Inverse Hamiltonian Reduction: A Lightning Intro

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$V^k(\mathfrak{g})$

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V^k(\mathfrak{g}) \arrow[d, "H^0_{DS}(-\text{,}\, d_f)"] \arrow[ddd, bend right] \\
W^k(\mathfrak{g}, f_1) \arrow[dd, bend right] \\
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W^k(\mathfrak{g}, f_{max})
\end{tikzcd}$$

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Key point: $W^k(\mathfrak{g},f_{max})$-Mod is finite and semisimple

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IHR: $V^k(\mathfrak{g}) \hookrightarrow W^k(\mathfrak{g}, f_i) \otimes FF$

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\begin{tikzcd}
V^k(\mathfrak{g})\text{-Mod} \arrow[d, "H^0_{DS}(-,d_f)"'] \arrow[ddd, bend right=60] \\
W^k(\mathfrak{g},f_1)\text{-Mod} \arrow[u, bend right] \arrow[dd, bend right] \\
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Lifts to (families of) functors

$$W^k(\mathfrak{g},f_i)\text{-Mod} \longrightarrow V^k(\mathfrak{g})\text{-Mod}$$

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- Ind/Res action on Mod Cats? $\rightsquigarrow$ Categorification?